Analysis of Dielectric Resonators with Tuning Screw and Supporting Structure

FELIX HERNÁNDEZ GIL AND JORGE PÉREZ MARTÍNEZ

Abstract —**The finite-element method is applied to calculate the resonant frequencies and electromagnetic field distributions of axisymmetric dielectric resonator modes. Analyses of several resonator shapes, including ring resonator, double resonator, and rod resonator with tuning screw and supporting structure, are made.**

I. Introduction

THE APPLICATION of dielectric resonators in microwave and millimeter circuits requires rigorous and efficient models to calculate resonant frequencies and field distributions. Several methods based in modal development have been presented to analyze rod [1], [2], ring [3], and multilayered cylindrical resonators [4]. However, these procedures are specifically designed to study determinated structures and may require modifications in their analysis procedures to study others.

Numerical methods can be applied to very different structures entering new data in a general computer program. Some of these are the finite-differences method [5], the surface integral equation formulation [6], and the differential method of Vincent [7].

In this paper, a numerical procedure based on the finite-element method to calculate the resonant frequencies of the TE and TM modes of axisymmetric dielectric resonators is presented. This procedure allows the study of the most common resonators (ring, rod, and double resonators) and different environments (supporting structures, tuning mechanisms), and can be also used to design new structures. This method also provides the field distributions useful to design coupling mechanisms and to calculate quality factors.

II. Method of Analysis

The configuration to be analyzed consists of an axisymmetric closed metal cavity with an arbitrary number of dielectrics and conductors inside it. A cylindrical dielectric resonator with a tuning screw and dielectric support (Fig. 1) is an example of this kind of structure.

A. Variational Formulation

A finite-element formulation has been made using the variational form proposed by Berk [8]. The solution of Maxwell's equations can be derived from the following dual-vectorial variational principles for electromagnetic fields:

$$F(\overline{\Psi}) = \int_{\Omega} p(\nabla \times \overline{\Psi})\cdot(\nabla \times \overline{\Psi})^* d\Omega - K_0^2 \int_{\Omega} q\overline{\Psi}\cdot\overline{\Psi}^* d\Omega$$

$$\overline{\Psi} = \begin{cases} \text{magnetic field}, & p = 1/\epsilon_r, \quad q = \mu_r \\ \text{electric field}, & p = 1/\mu_r, \quad q = \epsilon_r. \end{cases} \tag{1}$$

Considering only axisymmetric structures and modes (TE and TM), the two functionals can be reduced to scalar form using only the ϕ component of the field (electrical field for TE modes and magnetic field for the TM modes)

$$F(\Psi) = \int_{\Omega} p\left[\left(\frac{\partial \Psi}{\partial r}\right)^2 + \left(\frac{\partial \Psi}{\partial z}\right)^2 + \left(\frac{\Psi}{r}\right)^2 + 2\Psi\frac{\partial \Psi}{\partial r}\right] d\Omega - K_0^2 \int_{\Omega} q(\Psi)^2 d\Omega. \tag{2}$$

B. Discretization of the Problem

The domain is discretized into a finite number of subregions called elements. Permitivity inside each element must be constant. The size and shape of these regions are important parameters that can be adjusted to optimize the analysis efficiency.

The analysis of dielectric resonators requires generally smaller elements near and inside the resonator because the components of the fields there are more important than in other regions to calculate accurately the resonant frequency.

The fields are approximated over each element by polynomials that are defined using the values of the field in some points of the element. The field is expressed as

$$\left.\begin{matrix} E_\phi \\ H_\phi \end{matrix}\right\} = \sum_1^r x_i \cdot N_i \tag{3}$$

where

- N_i polynomials to approximate the field,
- r total number of nodes,
- x_i field value in the node i.

Substituting this approximate expression of the fields in the variational form and applying stationariness in the nodes using the Rayleigh–Ritz procedure, the following

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F. Hernández is with Dpto de Investigación y Desarrollo, Telefónica, 28020 Madrid, Spain.

J. Pérez is with the Microwave Department, E.T.S. Ing. Telecomunicación, 28040 Madrid, Spain.

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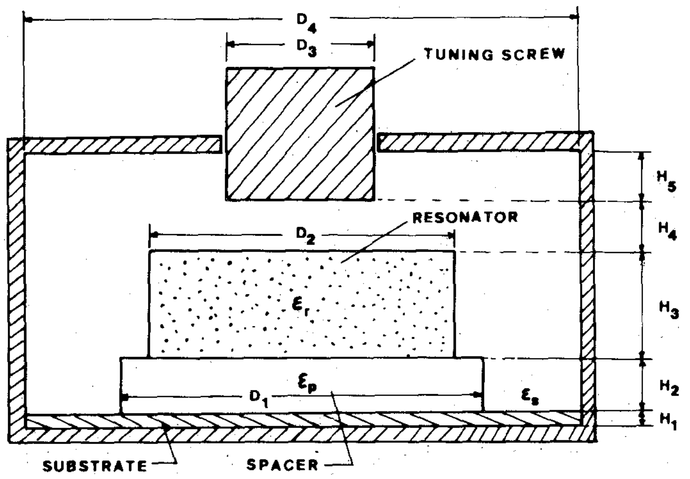


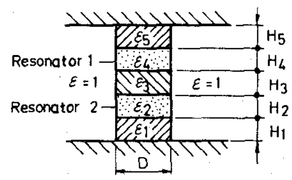
Fig. 1. Dielectric rod resonator with tuning screw and supporting structure.

TABLE I
CONVERGENGE OF THE METHOD FOR A DIELECTRIC ROD RESONATOR ON MICROSTRIP SUBSTRATE

Number of nodes by element	Number of elements	Number of equations	Frequency (GHz)	CPU (seconds)
4	30	20	10.8086	1
4	120	99	10.5968	2
4	480	437	10.5453	13
8	30	69	10.5307	2
8	120	317	10.5284	13
8	270	745	10.5283	42
9	30	99	10.5302	3
9	120	437	10.5284	19
9	270	1015	10.5283	63
12	30	118	10.5284	6
12	120	535	10.5283	38

$D_2 = 6.533$ mm, $D_4 = 1.5 * D_2$, $H_1 = 0.251$ mm, $H_2 = H_5 = 0$, $H_3 = 2.31$ mm, $H_4 = 1.5 * H_3$, $\epsilon_r = 37$, $\epsilon_s = 2.17$ (see Fig. 1 for definition of dimensions). The CPU times are on a digital VAX-11/750 computer.

TABLE II
DOUBLE DIELECTRIC RESONATOR



$H_2 = H_4 = 3.52$ mm
D= 14.98 mm
$\epsilon_2 = \epsilon_4 = 34.1$
$\epsilon_1 = \epsilon_3 = \epsilon_5 = 2.04$

DIMENSIONS (mm)			RESONANT FREQUENCY (GHz)		
H_1	H_3	H_5	Experimental	Method A	This method
1.92	3.48	5.93	4.182	4.180	4.184
1.92	3.48	2.48	4.433	4.420	4.423
2.48	1.4	4.48	4.061	4.043	4.047

Method A: S. Maj and M. Pospiezalski [4]. Experimental values from the same authors.

standard eigenvalue matrix equation is obtained:

$$[A]_{r\times r}\cdot[x]_{r\times 1} = \lambda[B]_{r\times r}\cdot[x]_{r\times 1}$$

$$A_{ij} = \int_\Omega \left(\frac{\partial N_i}{\partial r}\cdot\frac{\partial N_j}{\partial r} + \frac{\partial N_i}{\partial z}\cdot\frac{\partial N_j}{\partial z} + \frac{N_i\cdot N_j}{r^2} + N_i\frac{\partial N_j}{\partial r} + N_j\frac{\partial N_i}{\partial r}\right) d\Omega$$

$$B_{ij} = \left(\frac{2\pi}{c}\right)\int_\Omega qN_iN_j\,d\Omega. \tag{4}$$

The electric-field formulation requires the introduction of a perfect electric boundary condition on the metal walls. It is also useful to make the field zero in the symmetric axis for both electric and magnetic formulation. This reduces the size of the system and eliminates some spurious solutions.

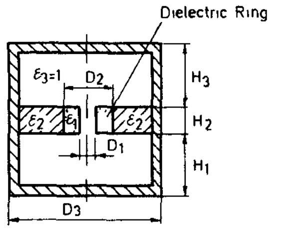
The system eigenvalues are the squares of the resonant frequencies, and the associate eigenvectors are the polynomial coefficients that describe the field distributions of the modes.

C. Computation

Most of the coefficients of this system are zero so it is convenient to use special matrix algorithms to solve it. The inverse iteration method is applied to solve the system if one eigenvalue is needed, or the subspace iteration method for several eigenvalues [10]. Numerical integration is used to calculate the coefficients of the system. This simplifies and adds flexibility to the program allowing an easy introduction of different elements and formulations.

Table I shows the convergence of the method for a dielectric rod resonator on a microstrip substrate using

TABLE III
DIELECTRIC RING RESONATOR

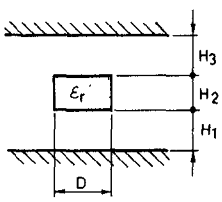


D_1=3.044 mm ϵ_1=37.5
D_2=9.501 mm ϵ_2=1.037
D_3=14.44 mm
H_1=H_3=4 mm
H_2=4.124 mm

	Method B	This method
Resonant frequency for the $TE_{0\gamma\delta}$ mode	6.0485 GHz	6.059 GHz
Resonant frequency for the $TM_{0\gamma\delta}$ mode	9.324 GHz	9.369 GHz

Method B: Mode-matching technique used by Y. Kobayashi and M. Miura [3].

TABLE IV
DIELECTRIC ROD RESONATOR



H_1=0
H_2=13.37 mm
D=8.995 mm
ϵ_r=34.61

DISTANCE TO THE TOP METAL WALL (mm)	RESONANT FREQUENCY (GHZ)	
H_3	Method C	This Method
9.03	2.916	2.907
33.03	2.886	2.861

Method C: Proposed by D. Maystre and P. Vincent [2].

several kinds of elements. Poor results are obtained with four-node linear elements because the convergence is very slow. Eight- and twelve-node elements provide better results. High numerical errors have been observed using elements of more than twelve nodes.

As can be seen in Table I, the method is always upper convergent to the real solution. This result has usually been observed for TE modes. This allows an easy determination of the right mesh increasing the density of each zone until a minimum in the resonant frequency is observed. This rule is broken if too high order elements are used. In this case, the values of the solutions are lower than the real ones and no convergence is achieved.

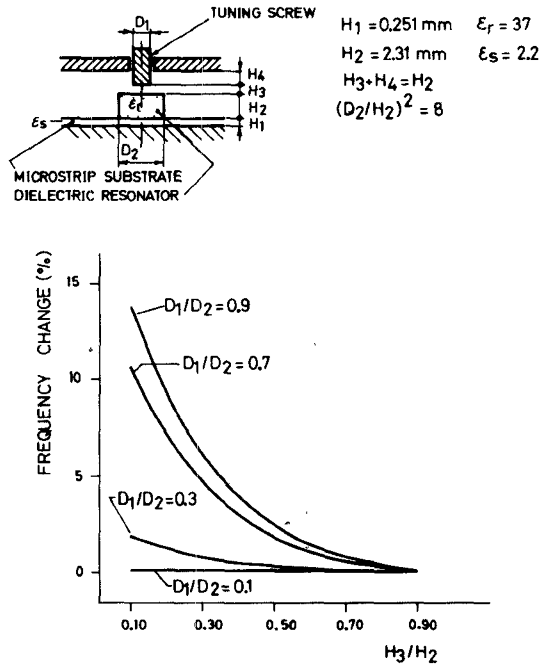


Fig. 2. Frequency change of a dielectric rod resonator for several tuning screw diameters.

The required computing time to solve the problem depends a lot on the complexity of the analyzed structure and on the desired precision. For example, to calculate the resonant frequency of a dielectric rod resonator on a microstrip substrate with convergence better than 1E-4 (118 equations, 18 percent of nonzero coefficients) takes about 5s on a digital VAX-11/750 computer. More complex structures like metal-film trimmed resonators [9] take about 20s.

III. RESULTS

A. Comparison with Other Theories

A comparison between the results obtained with this method for the resonant frequencies of the lowest TE mode and those obtained by others authors [2]–[4] with rigorous methods for the double resonator, ring resonator, and rod resonator are shown in the Tables II–IV. The differences are always smaller than one percent.

To study double dielectric resonators and rod dielectric resonators mounted on structures without lateral walls, the distance to those walls in the model is increased until no influence on the resonant frequency is observed. This does not involve too much computational cost for the lowest TE mode because the field decreases very quickly.

B. Tuning

Curves to design metal screws to adjust the resonant frequency of dielectric rod resonators have been calculated. This allows one to choose an adequate screw diameter to obtain the desired tuning margin, and is especially useful

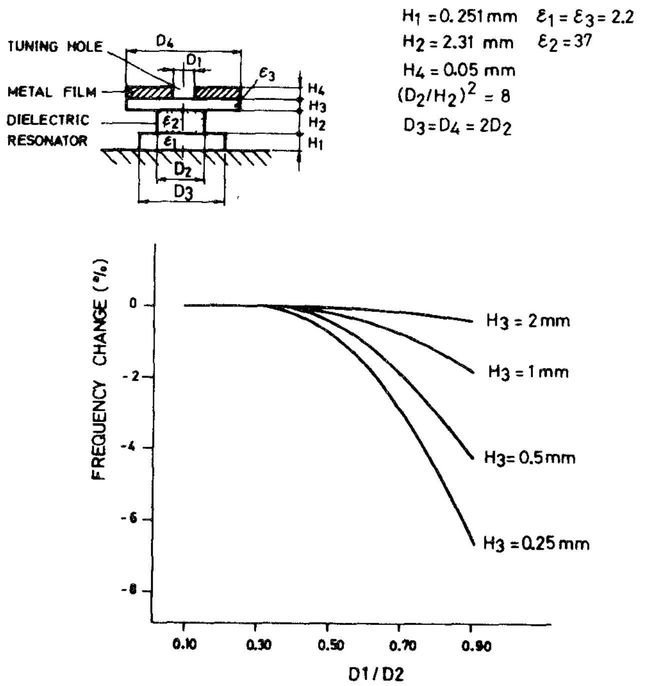


Fig. 3. Frequency change of a dielectric rod resonator with metal film tuning.

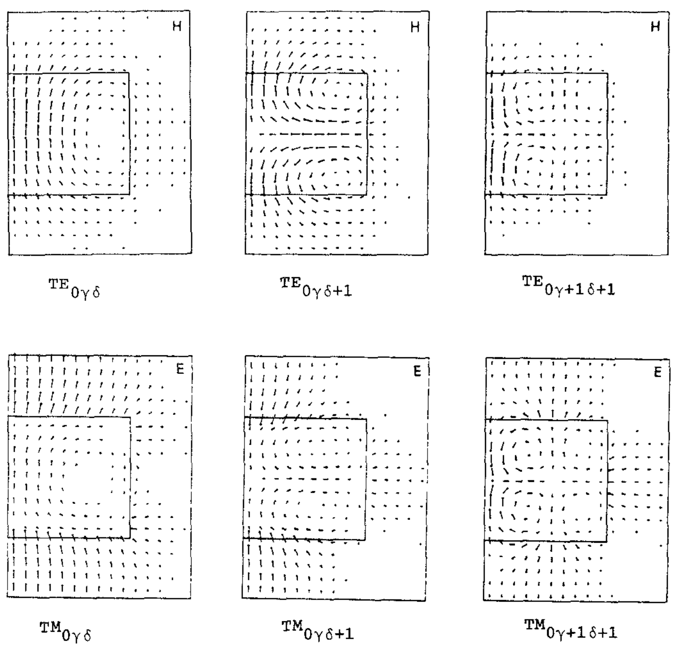


Fig. 4. Field distributions for several modes of a shielded dielectric rod resonator ($\epsilon_r = 36$).

to design filters where diameters must be small and the approximation of wide diameter of the screw compared with the resonator diameter is no longer valid. Fig. 2 shows one of these curves.

Another method to adjust the frequency is to change the area of a metal film situated on the resonator [9]. Curves for several distances between the resonator and the metal film are presented (Fig. 3). Film thickness has been taken into account. The study of this structure using model matching methods is difficult.

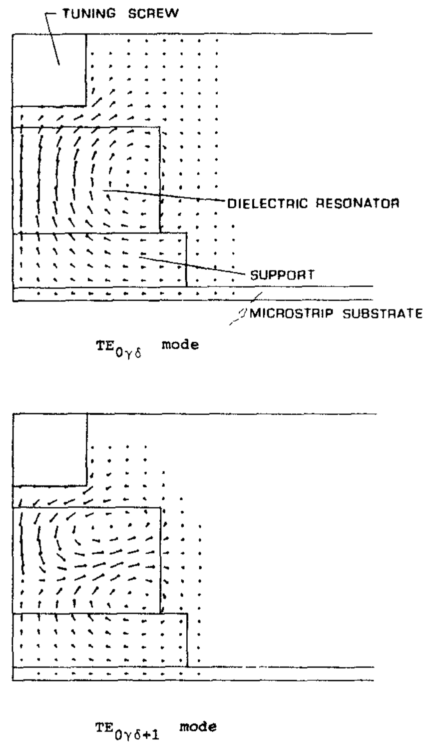


Fig. 5. Magnetic field distributions of a dielectric rod resonator with tuning screw and support.

C. *Field Distributions*

The eigenvectors associated with the eigenvalues provide the polynomial coefficients of (3) that describe the fields. Using Maxwell's equations, the other components of the field can be calculated.

Fig. 4 shows the field distributions of a shielded cylindrical dielectric resonator for several TE and TM modes. The field distributions inside and near the resonator are consistent with those obtained for isolated resonators using the integral surface equation formulation [11].

Fig. 5 shows the magnetic-field distribution of a dielectric rod resonator with a tuning screw and supporting disk for the $TE_{0\gamma\delta}$ and the $TE_{0\gamma\delta+1}$ modes. The arrows length is linearly proportional to the field amplitude in each point. Knowledge of the field distributions is useful to design a coupling mechanism and to suppress undesired modes.

IV. Conclusions

The finite-element method is a useful tool to calculate the TE and TM resonant frequencies and field distributions of axisymmetric dielectric resonators. Many kinds of structures can be studied using only one computer program with a moderate computing time and good precision.

This method can be especially interesting to study complex resonators and to design tuning devices and supporting structures.

Acknowledgment

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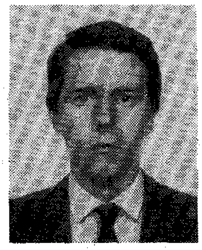
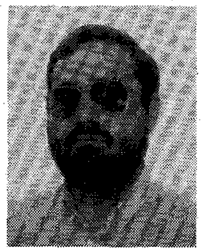


F. Hernández Gil was born in Salamanca, Spain, in 1957. He received the B.Sc. degree in telecommunication engineering from the Polytechnic University of Madrid in July 1981.

Since 1982, he has been with the Department of Microwaves of the Polytechnic University of Madrid, where he is working towards his Ph.D., in the study of dielectric resonators and other field problems using the finite-element method. In 1985, he joined the Research and Development Center of Telefonica. His current interest is the analysis of millimeter-wave and microwave circuits using numerical techniques.



J. Pérez Martínez was born in Castejón, Spain, in 1954. He obtained the B.Sc. and Ph.D. in telecommunication engineering from the Polytechnic University of Madrid and the B.Sc. in social sciences from the Complutense University of Madrid. He is a Professor in the Department of Microwaves at the Polytechnic University of Madrid. He is currently working on Gunn and GaAs MESFET oscillators, front-ends for direct broadcast satellite TV, and millimetre-wave integrated circuits.